A COMPARISON OF HIGH RATE ALGEBRAIC AND NON-ORTHOGONAL STBCS

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Abstract

We unify and compare two high rate space-time coding constructions and layering techniques for MIMO systems. Algebraic space-time coding constructions are revisited and their relation to non-orthogonal codes (with quasi-orthogonal layers) is established.

- **We discuss the class of perfect and golden space-time block codes, consisting of the version used in IEEE 802.16e specification for a system with 2 tx and 2 rx antennas.**
- **The main contribution of the paper is to consider both algebraic and non-orthogonal space-time codes for the 4 tx and 2 rx antenna setup.**

System Model

MIMO Model

- We consider a $n_T \times n_R$ MIMO system, where $n_T \geq n_R$
- Block fading assumption for channel matrix $\mathbf{H}$

$$\mathbf{Y}_{n_R \times T} = \mathbf{H}_{n_R \times n_T} \mathbf{X}_{n_T \times T} + \mathbf{Z}_{n_R \times T} \quad (1)$$

- We assume $\mathbf{Z} \sim \mathcal{CN}(0, N_0)$ and
- $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the independent Rayleigh fading channel matrix with complex i.i.d. entries $\mathcal{CN}(0, 1)$, which is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_{n_R} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,n_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1} & h_{n_R,2} & \cdots & h_{n_R,n_T} \end{bmatrix}.$$

The space-time codes are designed under the assumption that the elements of the channel matrix are Rayleigh distributed and they vary independently from one block to another.

TAST Codes

Threaded-algebraic space-time (TAST) codes are constructed by transmitting a scaled DAST code in each layer (or thread) l,

$$\mathbf{x}_l = \phi_l \mathbf{M} \mathbf{b}_l, \qquad (l = 1, \ldots, n_R) \quad (2)$$

where $\mathbf{x}_l$ are the encoded symbols, $\mathbf{b}_l$ are the complex QAM information symbol vectors, and ϕ_l is chosen to ensure full diversity and maximize the coding gain of the component codes. $\phi_l = \phi^{(l-1)/n_T}$, where $\phi = e^{i\lambda}$ is either algebraic or transcendental.

In (2), $\mathbf{M} \in \mathbb{C}^{n_T \times n_T}$ is a rotation matrix defining a DAST code, which is constructed from an algebraic number field $\mathbb{Q}(\theta)$ of degree n_T. Let $\mathbf{s} = [s_1, \ldots, s_{n_T}]^T = \mathbf{M}\mathbf{b}$ and $\hat{\mathbf{s}} = [\hat{s}_1, \ldots, \hat{s}_{n_T}]^T = \mathbf{M}\hat{\mathbf{b}}$ be two different DAST codewords, where $\mathbf{b}$ and $\hat{\mathbf{b}}$ are two different information symbol vectors. The rotation matrix $\mathbf{M}$ is chosen to maximize the associated minimum product distance $d_p(\mathbf{s}, \hat{\mathbf{s}})$. defined as

$$d_p = \min_{\mathbf{s} \neq \hat{\mathbf{s}}} \prod_{i=1}^{n_T} |s_i - \hat{s}_i| \, . \quad (3)$$

One can easily verify that DAST codes achieve full diversity, and their coding gains are proportional to the minimum product distance associated with the rotations used.

For L layers, where $L = n_R$ for the system in this paper, we can write the TAST codeword matrix as

$$\mathbf{X} = \sum_{l=1}^{n_R} \left(\phi_l \mathbf{e}^{l-1} \right) \mathrm{diag}\left(\mathbf{M}\mathbf{b}_l \right), \quad (4)$$

where

$$\mathbf{e} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & & 0 \\ 0 & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}. \quad (5)$$

Perfect STBCs

Perfect codes are full rate and full diversity $n_T n_R$. Furthermore they posses the non-vanishing determinant property that guarantees that they achieve the DMG tradeoff. The QAM information symbols are linearly encoded by such STBCs into an $n_T \times n_T$ codeword matrix $\mathbf{X} = \{x_{i,l}\} \in \mathbb{C}, i, l = 1, \ldots, n_T$. The perfect STBCs are constructed based on cyclic division algebras, where the codeword with L layers is given by

$$\mathbf{X} = \sqrt{\frac{n_T}{L}} \sum_{l=1}^{L} \mathbf{e}^{l-1} \mathrm{diag}\left(\mathbf{M}\mathbf{b}_l \right), \quad (6)$$

where

$$\mathbf{e} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & & 0 \\ 0 & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & 1 \\ \gamma & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (7)$$

and γ is chosen from $\mathbb{Z}[j]$ in order to achieve the full diversity and non-vanishing determinant. The factor $\sqrt{\frac{n_T}{L}}$ is a power normalization and assures transmit power the same total power is transmitted when not all layers are encoded. Comparing to TAST codes, we have a different $\mathbf{e}$ matrix and $\phi = 1, \gamma = j$.

In this paper, we only consider subcodes of the perfect STBCs with a reduced number of layers, i.e., $L = n_R = 2$ and the transmission matrix is

$$\mathbf{X} = \sqrt{\frac{n_T}{2}} \sum_{l=1}^{2} \mathbf{e}^{l-1} \mathrm{diag}\left(\mathbf{M}\mathbf{b}_l \right).$$

Note that with the Perfect code given above only two transmit antennas (out of four) are used at any time instant.

The unitary generator matrix $\mathbf{M}$ for 4×4 Perfect STBC is given in the paper.

Quasi-orthogonal codes

They induce quasi-orthogonal layers where only some symbols interfere with each other while others remain orthogonal. In a variant of Double ABBA ("DjABBA"), with $\mathbf{X}_A, \mathbf{X}_B, \mathbf{X}_C$ and $\mathbf{X}_D$ STTD blocks encoding pairs of symbols

$$\mathbf{X}^T = \begin{bmatrix} \cos\rho\, \mathbf{X}_A + \sin\rho\, \mathbf{X}_C & \cos\rho\, \mathbf{X}_B + \sin\rho\, \mathbf{X}_D \\ \jmath\,(\sin\rho\, \mathbf{X}_B - \cos\rho\, \mathbf{X}_D) & \sin\rho\, \mathbf{X}_A - \cos\rho\, \mathbf{X}_C \end{bmatrix} \quad (8)$$

Thus, the matrix transmits eight symbols using a modulation matrix of size 4×4, which is identical to that of TAST or Perfect codes with $L = 2$, given above. Due to STTD structure, by puncturing antennas 2 and 4 from $\mathbf{X}$, the result is

$$\begin{bmatrix} \cos\rho\, x_1 + \sin\rho\, x_5 & \cos\rho\, x_3 + \sin\rho\, x_7 \\ \cos\rho\, x_2 + \sin\rho\, x_6 & \cos\rho\, x_4 + \sin\rho\, x_8 \\ \jmath\,(\sin\rho\, x_3 - \cos\rho\, x_7) & \sin\rho\, x_1 - \cos\rho\, x_5 \\ \jmath\,(\sin\rho\, x_4 - \cos\rho\, x_8) & \sin\rho\, x_2 - \cos\rho\, x_6 \end{bmatrix}, \quad (9)$$

which is a redundant but equivalent representation of a particular 2×2 Golden code, provided that ρ is appropriately selected.

We see that TAST, Perfect, and Quasi-orthogonal codes are linked to each other. DABBA, in contrast to TAST, is known to achieve second order capacity of the 4×2 MIMO channel.

Performance

Uncoded

We evaluate the performance of selected designs using 4 transmit and 2 receive antennas, QPSK modulation, with 4 b/s/Hz. We compare DjABBA (with $\rho = \pi/4$) to a two layer Perfect code. Fig. 1 shows the result with Sphere and LMMSE detection in an iid Rayleigh fading channel. It is seen that DjABBA improves in perfect code by about 0.5 dB at high SNR. Although TAST result are not depicted, the simulations have shown that the Perfect code improves on TAST by about 0.5 dB at high SNR.

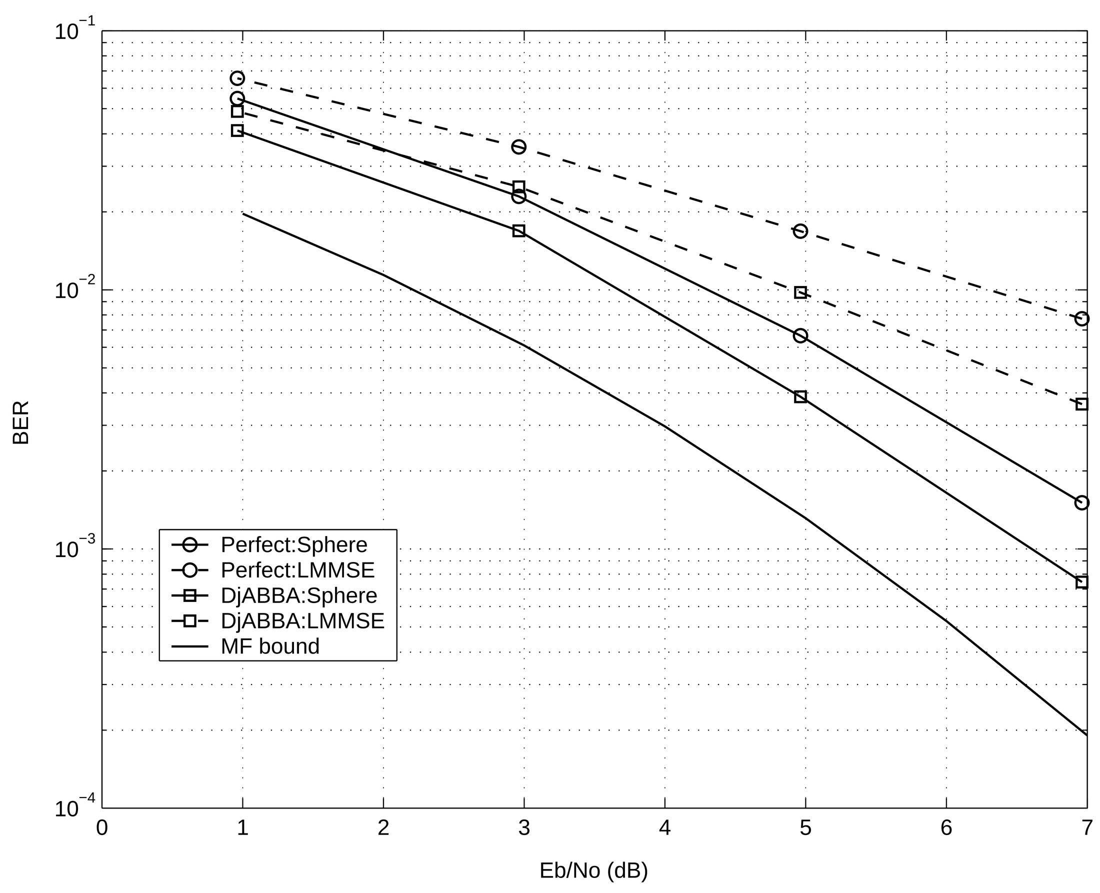


Fig. 1: Uncoded BER of a two layer Perfect code and DjABBA in 4 tx – 2 rx configuration and iid Rayleigh channel.

Coded

The performance evaluation for coded systems is carried out in an IEEE 802.16-2004 compliant WiMAX simulator. For the simulations we use the OFDM physical layer with an FFT size of 256. The standard-conform coding consists of a concatenation of an outer Reed-Solomon code and an inner convolutional code. A two stage interleaver after the encoder avoids error bursts caused by subcarriers with low SNR. The relevant WiMAX system parameters are summarized in Table **??**.

We evaluate the performance for spatially uncorrelated flat and frequency selective block fading channels. The frequency selective channel is generated according to the ITU Pedestrian B power delay profile. The receivers for the different space-time codes are maximum likelihood receivers with hard demapping. For the 4×2 systems the ML receiver is implemented as sphere decoder.

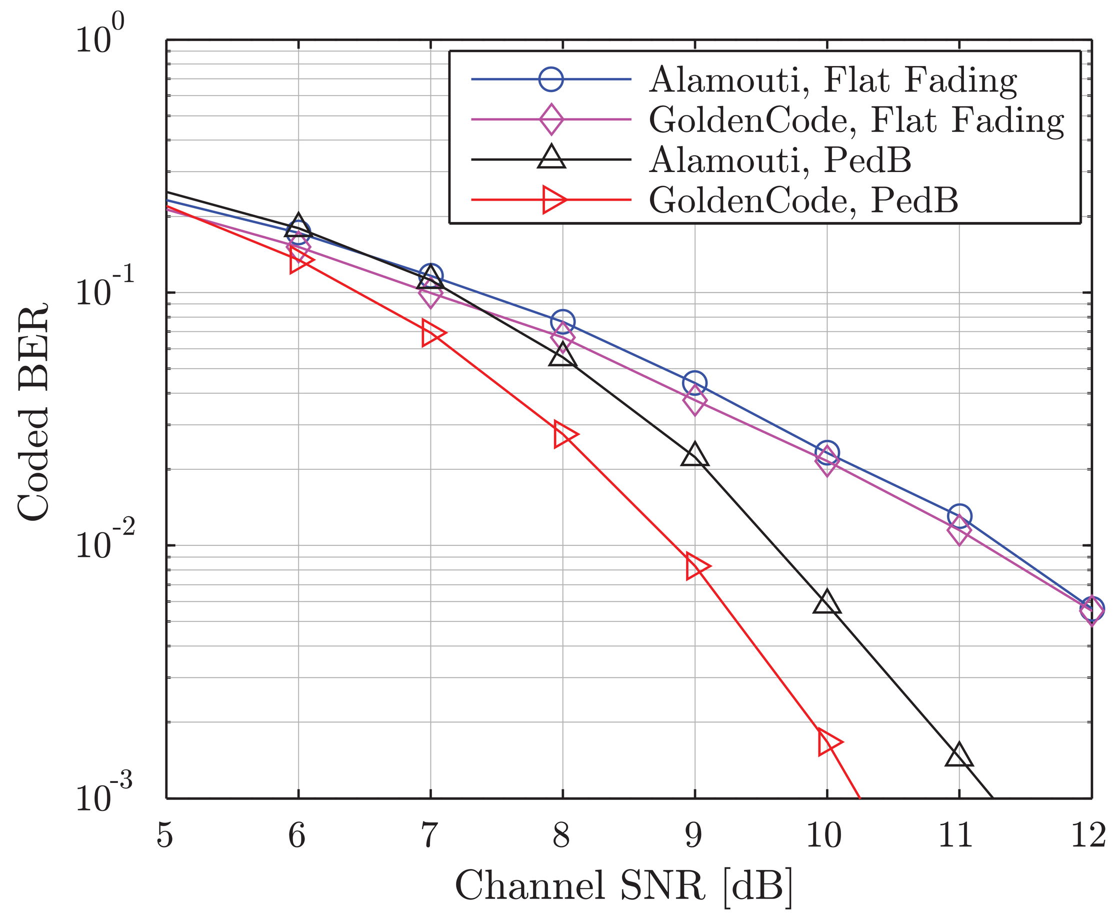
2×2 systems

The 2 tx and 2 rx antenna system is compared for Alamouti coded and Golden coded transmit signals. For Alamouti coding we employ 16-QAM modulation, and for Golden code 4-QAM to allow for a fair comparison. For both space-time codes the channel coding is exactly the same. The results in Fig. 2 show that the Golden Code enjoys approx. 0.8 dB gain over Alamouti in a Pedestrian B environment when combined with the WiMAX conformant concatenated Reed-Solomon-Convolutional code. In a flat fading environment however, the gain is only about 0.3 dB.

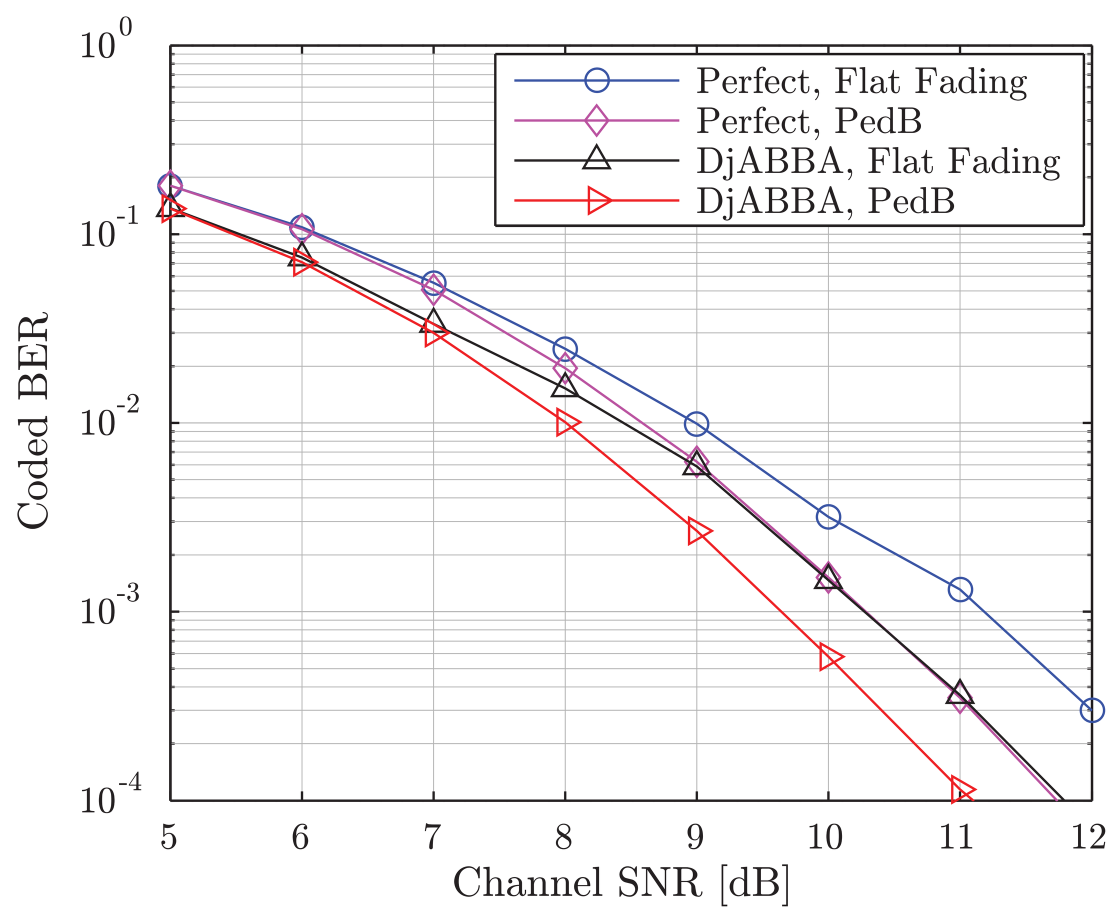
4×2 systems

A comparison of DjABBA (with optimal $\rho = 0.8881$ and the Perfect code with two encoded layers is shown in Fig. 2. Here, we averaged over 11000 channel realizations for the flat fading case and 6000 channel realizations for the Pedestrian B channel model. Channel coding and 4-QAM modulation is the same as for the 2×2 system employing the Golden space-time coding to allow for comparisons between the 4×2 and 2×2 systems. In both scenarios, flat fading and Pedestrian B, the DjABBA outperforms the Perfect code by 0.6 dB.

The weaker performance of the two-layer Perfect code is due to the reduced number of diversity branches available in one channel use. On the contrary, the DjABBA uses all independent branches between the 4 tx and the 2 rx antennas in each channel use.



Comparison of Golden Code and Alamouti for a coded 2x2 WiMAX transmission.



Comparison of DjABBA and Perfect code for a coded 4x2 WiMAX transmission.

Conclusion

We reviewed and evaluated two different high rate space-time coding concepts,

- an algebraic (Perfect) code and a
- quasi-orthogonal code (DjABBA).

Both code constructions are compared (design and performance). Performance is evaluated on a WiMAX compatible simulation chain. The comparisons are done for a medium rate service, QPSK, and coding rate 1/2. In this setup, DjABBA outperforms Perfect code by a fraction of a decibel. On the other hand, the Perfect code may have some implementation advantages in that at each time instant only two transmit antennas (out of four) are used.

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